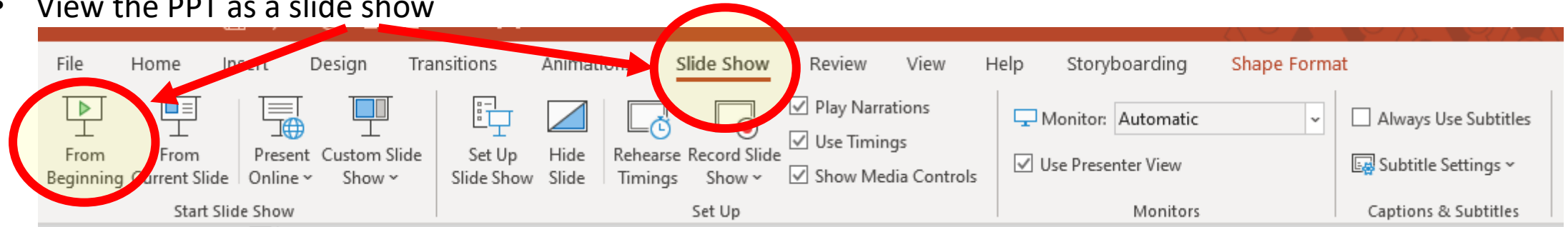


How to best use these slides...

- View the PPT as a slide show



- Then click through every step
 - Mouse clicks will advance the slide show
 - Left/right arrow keys move forward/backward
 - Mouse wheel scrolling moves forward/backward
- When a question is posed, stop and think it through, try to answer it yourself before clicking
- If you have questions, use PS discussion boards, email me, and/or visit us in a Teams class session!

LESSON 7. 5a

Solving Rational Equations

by Using the LCD

Today you will:

- Solve rational equations by using LCD
- Practice using English to describe math processes and equations

Core Vocabulary:

- Rational equation
- LCM – Lowest Common Multiple
- LCD – Lowest Common Denominator (LCM for the denominators)

Prior Vocabulary:

- Factor – one thing multiplied by another
- Term – one thing added to (or subtracted from) another

Factor = multiplied

- consider $a \cdot b + 1$
- a and b are factors

Term = added/subtracted

- consider $a \cdot b + 1$
- ab and 1 are terms

Review/Recap

- Yesterday we saw how to solve a rational equation using ***Cross Multiplication***
 - IMPORTANT:
 - Can only be a single fraction on the left...
 - ...and a single fraction on the right.
 - Multiple each side by the denominator from the other side.
- ***ALWAYS*** check your answer
 - Today we will see why!!!

When can we use **Cross Multiplication**?

- **ONLY** when the equation is one fraction equals another...
- ...when there is only one fraction on the left
- ...and only one fraction on the right
- Looks basically like this: $\frac{a}{b} = \frac{c}{d}$

So ... what do we if there is another term on either (or both) side?

- What if we have something like: $\frac{a}{b} + 1 = \frac{c}{d}$
- We **CANNOT** use cross multiplication ... because the left side has an extra term
- Wouldn't it be nice to just "get rid of" the denominators?
 - Isn't that basically what we're doing when we cross multiply?

Watch this...

$$\frac{15}{x} + \frac{4}{5} = \frac{7}{x} \rightarrow \underline{5x} \cdot \frac{15}{x} + \underline{5x} \cdot \frac{4}{5} = \underline{5x} \cdot \frac{7}{x} \rightarrow \cancel{5x} \cdot \frac{15}{\cancel{x}} + \cancel{5x} \cdot \frac{4}{\cancel{5}} = \cancel{5x} \cdot \frac{7}{\cancel{x}} \rightarrow 5 \cdot 15 + x \cdot 4 = 5 \cdot 7 \rightarrow 75 + 4x = 35$$

BUMMER!
Can't use cross
multiplication

multiply every **TERM**
by the LCD

simplify: in each **TERM**,
cancel common factors

now this is MUCH easier
to work with isn't it?

What is the LCD? $5x$

What did we do?

1. Found the LCD
2. Multiplied every term by the LCD
 - Note we did not multiply it out
 - ...we just put in the LCD, showing it multiplied to every term
- Cancelled common factors
- *THEN* did the multiplication for each term
- ...now just solve using normal algebra steps!

Solve each equation.

a. $\frac{5}{x} + \frac{7}{4} = -\frac{9}{x}$

b. $1 - \frac{8}{x-5} = \frac{3}{x}$

SOLUTION

a. $\frac{5}{x} + \frac{7}{4} = -\frac{9}{x}$

$$4x\left(\frac{5}{x}\right) + 4x\left(\frac{7}{4}\right) = 4x\left(-\frac{9}{x}\right)$$

$$20 + 7x = -36$$

$$7x = -56$$

$$x = -8$$

Write original equation.

Multiply each term by the LCD, $4x$.

Simplify.

Subtract 20 from each side.

Divide each side by 7.

Check

$$\frac{5}{-8} + \frac{7}{4} \stackrel{?}{=} -\frac{9}{-8}$$

$$-\frac{5}{8} + \frac{14}{8} \stackrel{?}{=} \frac{9}{8}$$

$$\frac{9}{8} = \frac{9}{8}$$



The solution is $x = -8$. Check this in the original equation.

b.

$$1 - \frac{8}{x-5} = \frac{3}{x}$$

Write original equation.

$$[x(x-5) \cdot 1] - \left[x(x-5) \cdot \frac{8}{x-5} \right] = x(x-5) \cdot \frac{3}{x}$$

Multiply each term by the LCD, $x(x-5)$.

$$x(x-5) - 8x = 3(x-5)$$

Simplify.

$$x^2 - 5x - 8x = 3x - 15$$

Distributive Property

$$x^2 - 16x + 15 = 0$$

Write in standard form.

$$(x-1)(x-15) = 0$$

Factor.

$$x = 1 \text{ or } x = 15$$

Zero-Product Property

► The solutions are $x = 1$ and $x = 15$. Check these in the original equation.

Check

$$1 - \frac{8}{1-5} \stackrel{?}{=} \frac{3}{1}$$

$$1 + 2 \stackrel{?}{=} 3$$

$$3 = 3 \quad \checkmark$$

Substitute for x

Note:
both solutions
worked here!

$$1 - \frac{8}{15-5} \stackrel{?}{=} \frac{3}{15}$$

$$1 - \frac{4}{5} \stackrel{?}{=} \frac{1}{5}$$

$$\frac{1}{5} = \frac{1}{5} \quad \checkmark$$

$$\text{Solve } \frac{6}{x-3} = \frac{8x^2}{x^2-9} - \frac{4x}{x+3}$$

SOLUTION

Write each denominator in factored form. The LCD is $(x+3)(x-3)$.

$$\frac{6}{x-3} = \frac{8x^2}{(x+3)(x-3)} - \frac{4x}{x+3}$$

$$\cancel{(x+3)(x-3)} \cdot \frac{6}{\cancel{x-3}} = \cancel{(x+3)(x-3)} \cdot \frac{8x^2}{\cancel{(x+3)(x-3)}} - \cancel{(x+3)(x-3)} \cdot \frac{4x}{\cancel{x+3}}$$

$$6(x+3) = 8x^2 - 4x(x-3)$$

$$6x + 18 = 8x^2 - 4x^2 + 12x$$

$$0 = 4x^2 + 6x - 18$$

$$0 = 2x^2 + 3x - 9$$

$$0 = (2x-3)(x+3)$$

$$2x-3=0 \quad \text{or} \quad x+3=0$$

$$x = \frac{3}{2} \quad \text{or} \quad x = -3$$

Check

Check $x = \frac{3}{2}$:

$$\frac{6}{\frac{3}{2} - 3} \stackrel{?}{=} \frac{8\left(\frac{3}{2}\right)^2}{\left(\frac{3}{2}\right)^2 - 9} - \frac{4\left(\frac{3}{2}\right)}{\frac{3}{2} + 3}$$

$$\frac{6}{-\frac{3}{2}} \stackrel{?}{=} \frac{18}{-\frac{27}{4}} - \frac{6}{\frac{9}{2}}$$

$$-4 \stackrel{?}{=} -\frac{8}{3} - \frac{4}{3}$$

$$-4 = -4 \quad \checkmark$$

Check $x = -3$:

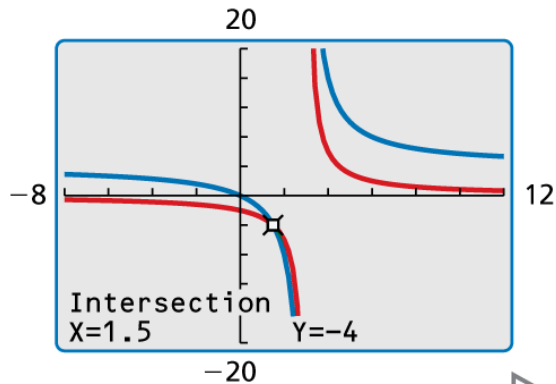
$$\frac{6}{-3 - 3} \stackrel{?}{=} \frac{8(-3)^2}{(-3)^2 - 9} - \frac{4(-3)}{-3 + 3}$$

$$\frac{6}{-6} \stackrel{?}{=} \frac{72}{0} - \frac{-12}{0} \quad \times$$

Division by zero is undefined.

ANOTHER WAY

You can also graph each side of the equation and find the x -value where the graphs intersect.



► The apparent solution $x = -3$ is extraneous. So, the only solution is $x = \frac{3}{2}$.

Note:

One of the *possible* solutions did **NOT** work here!
ALWAYS check your answers!!!

Review/Recap – Solving Rational Equations

Starting with a few duhs ... sorry:

- A rational expression is one polynomial divided by another
- A rational equation is an equation (has an equals sign) that includes rational expression(s)
- Factor: one thing that is multiplied by another
- Term: one thing that is added to/subtracted from another

We have two techniques for solving Rational Equations:

1. Cross multiplication

- Only works if just one fraction on each side: $\frac{a}{b} = \frac{c}{d}$
- Multiply each side by the denominator of the other

2. LCD

- Use if can't use cross multiplication
- Multiply every term by the LCD
- Cancel common factors
- Combine, simplify, solve!

Homework

Pg 396, #15-30